Scheduling Sport Leagues using Branch-and-Price

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Sports league scheduling is a hard combinatorial optimization problem. There is a vast field of requests arising in real world problems, e.g., organizational, attractiveness and fairness constraints. A single round robin tournament (SRRT) can be described as a league of a set T of n teams (n even) to be scheduled such that each team plays exactly once against each other team and such that each team plays exactly once per matchday (MD) resulting in a set P of $n-1$ MDs. Matches are carried out at one of opponents' stadiums. A team playing twice at home or twice away in two consecutive periods is said to have a break in the latter of both periods. The number of breaks is to be minimized. It is well known that at least $n-2$ breaks must occur. We focus on schedules having the minimum number of breaks. Costs corresponding to each possible match are given and the objective is to minimize the sum of matches' cost. This can be formulated as a cost minimization IP as follows:

$$\min \sum_{p\in P}\sum_{i\in T}\sum_{j\in T:j\neq i} c_{i,j,p}x_{i,j,p} \tag{1}$$

s.t.

$$\sum_{p\in P}(x_{i,j,p}+x_{j,i,p})=1 \qquad \forall i,j\in T:i<j \tag{2}$$

$$\sum_{j\in T:j\neq i}(x_{i,j,p}+x_{j,i,p})=1 \qquad \forall i\in T,p\in P \tag{3}$$

$$\sum_{j\in T:j\neq i}\left(x_{i,j,(p-1)}+x_{i,j,p}\right)-br_{i,p}\leq 1 \qquad \forall i\in T,p\in P^{\geq 2} \tag{4}$$

$$\sum_{j\in T:j\neq i}\left(x_{j,i,(p-1)}+x_{j,i,p}\right)-br_{i,p}\leq 1 \qquad \forall i\in T,p\in P^{\geq 2} \tag{5}$$

$$\sum_{i\in T}\sum_{t\in P^{\geq 2}} br_{i,t}\leq n-2 \tag{6}$$

$$x_{i,j,p}\in\{0,1\}\forall i,j\in T:j\neq i,p\in P \tag{7}$$

$$br_{i,p}\in\{0,1\}\forall i\in T,p\in P^{\geq 2} \tag{8}$$

$x_{i,j,p}$ is equal to 1 if and only if team $i\in T$ plays at home against team $j\in T$ at MD $p\in P$. Constraints (2) and (3) force the matches to form a SRRT. Equations (4) and (5) set $br_{i,p}$ to 1 if team i plays twice at home and away,

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respectively, at MDs $p-1$ and p. The overall number of breaks is restricted to be no more than $n-2$ by constraint (6). The objective function (1) represents the goal to minimize the overall sum of cost of all matches. Costs can be interpreted in an abstract way here but it is not difficult to think of several applications having practical relevance. For example $c_{i,j,p}$ can be employed to represent the teams' neglected preferences to play home or away in p if match (i, j, p) is carried out. There are several papers covering models being equal or at least closely related to the one at hand, see [8], [3], [1], [7], and [4] for example. In [2] a basic SRRT problem covering (1), (2), (3), and (7) is proven to be NP-hard.

We exhibit an approach employing the well-known concept called column generation (CG) which enumerates the variables of a large-scale linear program implicitely. CG has been successfully implemented for graph coloring in [5]. Scheduling a SRRT is equivalent to edge-coloring a complete graph K_n with $n-1$ colors. Our approach considers MDs as columns resulting in the pricing problem being a standard perfect-matching problem. Furthermore, we take home-away patterns (HAP) into account. A HAP can be represented by a string for each team i containing 0 in slot p iff i plays home at MD p. If we assign a HAP to each single team the pricing problem can be reduced to 2-dimensional assignment problems which can be solved efficiently by, e.g., the hungarian method.

This CG approach is employed within a branch-and-price framework. The branching concept underlies the idea to create sets of HAPs by branching on break-periods of a specific team. There are several properties of HAPs inducing a minimum number of breaks known from [6] which can be employed to reduce the number of branches to be evaluated.

We present the integer program representing the structural requirements of SRRTs and several additional constraints. Furthermore, we give possible extensions considering availability of stadiums, attractive matches, and fairness aspects. Details of the CG approach are outlined as well as the branching concept. Finally, we show that our algorithm outperforms CPLEX 9.0 by means of computational results and propose fields of future research.

References

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