Constructive Algorithms for the Constant Distance Traveling Tournament Problem

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1 Constant Distance Traveling Tournament Problem

In this abstract, we deal with the Constant Distance Traveling Tournament Problem (CDTTP) [4], which is a special class of the Traveling Tournament Problem (TTP), established by Easton, Nemhauser and Trick [1]. We propose a lower bound of the optimal value of CDTTP, and two algorithms that produce feasible solutions whose objective values are close to the proposed lower bound. For some size of instances, our algorithms yield feasible solutions better than the previous best solutions.

In the following, several definitions for CDTTP are introduced. Given even n, the number of teams, a double round-robin tournament is a set of games in which every team plays every other team exactly once at home and once at away. A game is specified by an ordered pair of opponents. Exactly $2(n-1)$ slots or time periods are required to play a double round-robin tournament. Each team begins at its home site and travels to play its games at the chosen venues. Each team then returns (if necessary) to its home at the end of the schedule. The number of trips of a team is defined by the number of moves of the team between team sites. Consecutive away games for a team constitute a road trip; consecutive home games are a home stand. The length of a road trip or home stand is the number of opponents playing against in the road trip/home stand. The problem CDTTP is defined as follows.
Input: the number of teams, n;
Output: a double round-robin tournament of n teams such that

1. the length of any home stand and that of any road trip is at most three;
2. no repeaters (A at B immediately followed by B at A is prohibited);

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3. the total number of trips taken by teams is minimized.

The CDTTP and its variations are discussed in [3, 5]. The CDTTP can be considered as a special class of the original TTP [1] such that all distances between team sites are one.

In the rest of this abstract, a schedule of double round-robin tournament satisfying the above conditions 1 and 2 is called a *feasible schedule*.

2 Lower Bound

We proved the following lemma that provides a lower bound of the optimal value of CDTTP. Due to space limitation, the proof is omitted.

Lemma 1. *The total number of trips of every feasible schedule of* n *teams is greater than or equal to* $\mathrm{LB}(n)$ *defined by*

$$\mathrm{LB}(n) \stackrel{\text{def.}}{=} \begin{cases} (4/3)n^2 - n & (n \equiv 0 \mod 3), \\ (4/3)n^2 - (5/6)n - 1 & (n \equiv 1 \mod 3), \\ (4/3)n^2 - (2/3)n & (n \equiv 2 \mod 3). \end{cases}$$

3 Algorithms

We propose two algorithms for constructing feasible schedules by modifying single round-robin tournaments. Due to space limitation, for both algorithms we describe these procedures for the case of $n \equiv 1 \mod 3$, and show only the results for $n \in \{0, 2\} \mod 3$.

3.1 Modified Circle Method

First, we propose the algorithm named *Modified Circle Method* (MCM). Denote the set of teams by $T = \{1, 2, \ldots, n\}$. We introduce a directed graph $G^{\mathrm{e}} = (T, A^{\mathrm{e}})$ with a vertex set T and a set of mutually disjoint directed edges

$$\begin{aligned} A^{\mathrm{e}} \stackrel{\text{def.}}{=} & \{(j, n+1-j) \ : \ \lceil j/3 \rceil \text{ is even}, \ 1 \leq j \leq n/2\} \\ & \cup \{(n+1-j, j) \ : \ \lceil j/3 \rceil \text{ is odd}, \ 1 \leq j \leq n/2\}. \end{aligned}$$

For any permutation π on T, $G^{\mathrm{e}}(\pi)$ denotes the set of $n/2$ matches satisfying that every directed edge $(u, v) \in A^{\mathrm{e}}$ corresponds to a match between $\pi(u)$ and $\pi(v)$ held at the home of $\pi(v)$. For each $j \in \{1, 2, \ldots, n-1\} = T \setminus \{n\}$, we define a permutation π^j by $(\pi^j(1), \pi^j(2), \ldots, \pi^j(n)) = (j, j+1, \ldots, n-1, 1, \ldots, j-1, n)$. Let G^{o} be a directed graph obtained from G^{e} by reversing the direction of the edge between 1 and n. Let X be a single round-robin tournament satisfying that matches in slot s are defined by $G^{\mathrm{o}}(\pi^s)$ (if $s \in \{1, 2, 3\} \mod 6$) and $G^{\mathrm{e}}(\pi^s)$ (if $s \in \{4, 5, 0\} \mod 6$). For each $i \in \{1, 2, \ldots, (n-1)/3\}$, we denote a partial schedule of X consisting of a sequence of three slots $(3i-2, 3i-1, 3i)$ by X_i. Now

we construct a feasible schedule Y by concatenating partial schedules consisting of three slots as follows:
$Y = (X_1, \overline{X_1}, \overline{X_2}, X_2, X_3, \overline{X_3}, \overline{X_4}, X_4, X_5, \ldots, X_{\frac{n-1}{3}}, \overline{X_{\frac{n-1}{3}}})$, where $\overline{X_i}$ is a partial schedule obtained from X_i by reversing venues.

For $n \equiv 1 \bmod 3$, MCM produces a feasible schedule Y of which the total number of trips is $(4/3)n^2 - (1/2)n - 4/3 = \mathrm{LB}(n) + (1/3)n - 1/3$. Since we do not have the space for describing MCM for $n \in \{0, 2\} \bmod 3$, we simply state our results as below. Let the total number of trips of a feasible schedule Y be $w(Y)$.

Theorem 1. *The Modified Circle Method produces a feasible schedule* Y *such that*

$$w(Y) = \begin{cases} (4/3)n^2 - (2/3)n - 1 \quad = \mathrm{LB}(n) + (1/3)n - 1 & (n \equiv 0 \mod 3), \\ (4/3)n^2 - (1/2)n - 4/3 = \mathrm{LB}(n) + (1/3)n - 1/3 & (n \equiv 1 \mod 3), \\ (4/3)n^2 + (1/6)n - 5/3 = \mathrm{LB}(n) + (5/6)n - 5/3 & (n \equiv 2 \mod 3). \end{cases}$$

3.2 Minimum Break Method

Here we propose the algorithm named *Minimum Break Method* (MBM). The procedure of MBM is also described only for the case of $n \equiv 1 \bmod 3$.

Given a feasible schedule, it is said that a team has a *break* at slot s if it has two consecutive home games (home break) or two consecutive away games (away break) in slots $s - 1$ and s. The total number of breaks $b(Y)$ is defined as the sum of the number of breaks of all the teams in a feasible schedule Y.

Let X be a schedule of a single round-robin tournament satisfying the following conditions:
(C1) the number of breaks $b(X)$ is equal to $n - 2$;
(C2) at each slot $s \in \{3, 5, 0\} \bmod 6$, exactly two teams have a break.
When $n \leq 50$, we have obtained a single round-robin tournament satisfying (C1) and (C2) by solving integer programming problems (e.g., see [2]). Now we construct a single round-robin tournament X' from X by reversing venues for each even slot. Then X' satisfies that exactly two teams have $n - 2$ breaks, other teams have $n - 3$ breaks, and every team has a break at slot s satisfying $s > 1$ and $s \equiv 1 \bmod 3$. For each $i \in \{1, 2, \ldots, (n-1)/3\}$, we denote a partial schedule of X' consisting of a sequence of three slots $(3i - 2, 3i - 1, 3i)$ by X'_i. Now we construct a feasible schedule Y' by concatenating partial schedules consisting of three slots as follows:
$Y' = (X'_1, \overline{X'_1}, X'_2, \overline{X'_2}, X'_3, \overline{X'_3}, X'_4, \overline{X'_4}, X'_5, \ldots, \overline{X'_{\frac{n-1}{3}}})$, where $\overline{X'_i}$ is a partial schedule obtained from X'_i by reversing venues.

For $n \equiv 1 \bmod 3$, the above procedure produces a feasible schedule Y' such that $w(Y') = (4/3)n^2 - (5/6)n - 1 = \mathrm{LB}(n)$. For $n \in \{0, 2\} \bmod 3$, again we simply state our results as follows.

Table 1. Results for $16 \leq n \leq 24$

n	LB(n)	MCM	MBM	known
16	327	332	*327	327
18	414	419	426	418
20	520	535	529	521
22	626	633	*626	632
24	744	751	755	757

*: our solutions that attain the lower bound LB(n)
known: the known best solutions in [4], as of April 2006

Theorem 2. *If there is a round-robin tournament satisfying Conditions (C1) and (C2), the Minimum Break Method produces a feasible schedule Y' such that*

$$w(Y') = \begin{cases} (4/3)n^2 - (1/2)n - 1 = \mathrm{LB}(n) + (1/2)n - 1 & (n \equiv 0 \mod 3), \\ (4/3)n^2 - (5/6)n - 1 = \mathrm{LB}(n) & (n \equiv 1 \mod 3), \\ (4/3)n^2 - (1/6)n - 1 = \mathrm{LB}(n) + (1/2)n - 1 & (n \equiv 2 \mod 3). \end{cases}$$

As mentioned before, we have already obtained schedules satisfying (C1) and (C2) for $n \leq 50$. Using MBM with them as initial solutions, we obtained feasible schedules (see Table 1).

Lastly, we summarize our results. For $n \equiv 0 \mod 3$, MCM gives better solutions compared to MBM. In contrast, for $n \in \{1, 2\} \mod 3$ MBM performs better though it needs an initial schedule satisfying Constraints (C1) and (C2). In addition, when $n \equiv 1 \mod 3$, with an initial schedule MBM yields a solution that attains LB(n), i.e., an optimal solution. Table 1 shows the results for $16 \leq n \leq 24$: for $n = 24$ both algorithms produced better solutions than the previous best; for $n = 22$ MBM gave an optimal solution.

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