# Referee Assignment in Sports Tournaments 

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## 1 Introduction

Optimization in sports is a field of increasing interest. Some applications have been reviewed by Ribeiro and Urrutia [7]. Combinatorial optimization techniques have been applied e.g. to the traveling tournament problem [3, 9], to playoff elimination in championships [8], and to the scheduling of a college basketball conference [6]. Easton et al. [2] reviewed scheduling problems in sports.

A common problem found in sports management is the assignment of referees to games already scheduled. The number of referees to be assigned to each game may vary depending on the sport or the league. For example, soccer games usually require three referees, while basketball games require only two. There are a number of rules and objectives that should be taken into account when referees are assigned to games. Games in higher divisions may require higher-skilled referees. Since referees may officiate several games during the day in amateur leagues, travel feasibility and travel times between the facilities where the games take place have to be considered. Additionally, and especially in some amateur children leagues, some of the referees are also players (or players' relatives). In this case, a natural constraint is that a referee cannot officiate a game that he/she is scheduled to play.

Referee assignment problems in other contexts have been addressed in [4, 5, 10]. Dinitz and Stinson [1] considered a problem involving referee assignment to tournament schedules, connecting room squares and balanced tournament designs. In this work, we address a simplified version of a referee assignment problem common to many amateur leagues of sports such as soccer, basketball, and baseball, among others. In the next section, we describe the main constraints and the objective function of the problem. The proposed solution strategy is summarized in Section 3. Concluding remarks and further extensions of this work are reported in the last section.

## 2 Problem statement

We consider the general problem, in which each game has a number of refereeing positions to be assigned to referees. The games are previously scheduled
and the facilities and time slots where they take place are known beforehand. In our approach, referees are assigned to empty refereeing positions, not to games. This approach allows not only to handle referee assignment problems in different sports, but also problems in tournaments where different games may need different numbers of referees. Games with pre-assigned referees to some refereeing positions can also be handled by this approach. Each refereeing position to be filled by a referee is called a referee slot.

Each referee slot has to be filled by a referee with a minimum skill level, which is previously determined and often related to the tournament division. Usually, a division corresponds to a set of teams formed by players under a certain age and with the same gender, e.g. boys under 16 years old. Each referee has a certain skill level defining the games he/she can officiate. Additionally, referees may declare their unavailability to officiate at certain time slots. Furthermore, each referee establishes the maximum number of games he/she is able to officiate and the target number of games he/she is willing to officiate. Travels are not allowed, i.e. referees that officiate more than one game in the same day must be assigned to games that take place at the same facility.

The Referee Assignment Problem (RAP) consists in assigning referees to all referee slots associated to games scheduled to a given time interval (typically, a day or a weekend), minimizing the sum over all referees of the absolute value of the difference between the target and the actual number of games assigned to each referee and satisfying a set of hard constraints listed below:

- all referee slots must be filled for all games;
- referees cannot officiate games in referee slots overlapping time slots where they are already scheduled to play or to officiate;
- referees cannot officiate games in referee slots where they declared to be unavailable;
- referees must meet the minimum skill level established for each referee slot;
- referees cannot officiate more than a given maximum number of games; and
- referees cannot officiate in two or more different facilities on the same day.


## 3 Solution strategy

The problem described in the previous section was formulated by integer programming. Only small instances with up to 40 games and 40 referees could be exactly solved by a commercial solver such as CPLEX 9.0.

We propose a three-phase heuristic approach to tackle real-life large instances of the referee assignment problem:

1. Apply a greedy heuristic to find an initial solution, possibly violating some constraints.
2. Make the initial solution feasible, by using a local search repair heuristic based on swap moves (referees assigned to two referee slots are swapped), exchange moves (the referee assigned to a referee slot is replaced by another referee), and group perturbations (all referee slots assigned to two referees officiating at different facilities are swapped).
3. Improve the feasible solution built in the previous step, by using a local search procedure based on exchange moves, simple perturbations (swap moves that do not change the value of the objective function), and group perturbations.

Randomly generated instances following patterns similar to real-life applications have been used in the computational experiments. They have up to 500 games and up to 1250 referees, different numbers of facilities, and different patterns of the target number of games each referee is willing to officiate. Preliminary experiments on a standard Pentium IV processor have shown that the greedy heuristic was able to build feasible solutions for the largest instances in less than 0.1 second. When a feasible solution was not found after the first phase, the local search repair heuristic took less than 1.0 second to found one. The third phase typically improved the feasible initial solutions by $50 \%$.

## 4 Extensions

We are currently working on extensions addressing further constraints of real-life applications. One of such extensions is the existence of hard or soft links between some referees. In this case, some referees may want to work with the same referees as partners in every game they officiate. This is the case when they are more confident to officiate together, but also when they want to travel in car pools or to officiate with relatives. In some situations, some referees can be unable to travel. Moreover, managers may want assignments matching preferences regarding the facilities, divisions, and time slots where the referees officiate.

Another extension occurs when referees are able to officiate games in different facilities. In this case, travel times between facilities should also be considered for feasibility matters. They can also be incorporated into the objective function, so as that the minimization of the total traveling time turns out to be another objective. The minimization of the waiting times between consecutive games assigned to the same referee is also relevant.

The referee assignment problem has clearly the flavor of a multi-criteria optimization application and we are also addressing the use of multi-criteria methods coupled with decision support systems for its solution in practice.

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